III Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (CBCS) (F+R) (2015-16 and Onwards)
MATHEMATICS – III

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

1. Answer **any five** questions : (5×2=10)

a) Find the number of generators of the cyclic group of order 30.

b) Define right coset and left coset of a group.

c) Show that the sequence $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.

d) State Raabe's Ratio test for convergence.

e) Test the convergence of the series :

$$1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}+....$$

f) Verify Rolle's theorem for the function $f(x)=x^2-6x+8$ in [2, 4].

g) State Cauchy's mean value theorem.

h) Evaluate $\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$.

PART – B

Answer **one full** question : (1×15=15)

2. a) If 'a and x' are any two elements of a group G then prove that $0(a) = 0(x\,a\,x^{-1})$.

b) Let G be a cyclic group of order d and 'a' be a generator, then prove that the element $a^k (k < d)$ is also a generator of G if and only if (k, d) = 1.

c) State and prove Fermat's theorem for groups.

OR

3. a) Prove that if ‘a’ is any element of a group G of order n then $a^m = e$ for any integer m if and only if n divides m.

b) Prove that every sub group of a cyclic group is cyclic.

c) Prove that every group of order less than or equal to 5 is abelian.

PART – C

Answer **two full** questions : **(2×15=30)**

4. a) Prove that the sequence $\left\{\dfrac{2n-7}{3n+2}\right\}$

i) is monotonically increasing

ii) is bounded.

b) Prove that a monotonic increasing sequence bounded above is convergent.

c) Show that the sequence $\{x_n\}$ where $x_1 = 1$ and $x_n = \sqrt{2 + x_{n-1}}$ is convergent and converges to 2.

OR

5. a) Show that $\{a_n\} = \dfrac{1}{1!} + \dfrac{1}{2!} + \dfrac{1}{3!} + \dots + \dfrac{1}{n!}$ is convergent.

b) Discuss the nature of the sequence $\left\{x^{1/n}\right\}$, $x > 0$.

c) Examine the convergence of the sequences :

i) $\dfrac{(n+1)^{n+1}}{n^n}$

ii) $\left\{\dfrac{2n^2+3n+5}{n+3}\right\} \sin\left(\dfrac{\pi}{n}\right)$.

6. a) State and prove D’Alemberts Ratio test for series of positive terms.

b) Test the convergence of the series $1 + \dfrac{1}{2} + \dfrac{1.3}{2.4} + \dfrac{1.3.5}{2.4.6} + \dots$

c) Sum the series to infinity $\frac{1}{5}-\frac{1.4}{5.10}+\frac{1.4.7}{5.10.15}-\frac{1.4.7.10}{5.10.15.20}+....$

OR

7. a) State and prove Cauchy's Root test for the convergence of series of positive terms.

b) Test the convergence of the series $\sum \frac{1.2.3.....n}{3.5.7.9.....(2n+1)}$.

c) Sum the series to infinity $\frac{1}{6}+\frac{1.4}{6.12}+\frac{1.4.7}{6.12.18}+....$

PART – D

Answer **one full** question : (1×15=15)

8. a) Prove that a function, which is continuous in a closed interval, takes every value between its bounds at least once.

b) Evaluate $\lim_{x\to 0}\frac{e^{1/x}}{1+e^{1/x}}$

c) Evaluate $\lim_{x\to 0}(1+\sin x)^{\cot x}$.

OR

9. a) Examine the differentiability of the function $f(x)=\begin{cases}x^2-1; & \text{for} \quad x\geq 1\\ 1-x; & \text{for} \quad x<1\end{cases}$ at x = 1.

b) State and prove Lagranges Mean value theorem.

c) Expand the function $\log_e(1+x)$ up to the term containing x^4 by Maclaurin's expansion.

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