VI Semester B.A./B.Sc. Examination, May/June 2018
(CBCS) (2016-17 and Onwards) (Semester Scheme) (Fresh + Repeaters)
MATHEMATICS – VIII

Time : 3 Hours Max. Marks : 70

***Instruction** : Answer **all** the questions/Parts.*

PART – A

Answer any five questions : (5×2=10)

1. a) Evaluate $\lim\limits_{z\to -i}\dfrac{z^2+1}{z^6+1}$.

   b) Prove that $u=\dfrac{1}{2}\log(x^2+y^2)$ is harmonic.

   c) Define an analytic function and give an example.

   d) Define bilinear transformation.

   e) Show that f(z) = cosz is analytic.

   f) State Liouvilles' theorem.

   g) Find the real root of the equation $x^3-9x+1=0$ in (2.9, 3) by bisection method.

   h) Using Newton-Raphson method, find the real root of $x^2+5x-11=0$ in (1, 2) in one iteration only.

PART – B

Answer four full questions : (4×10=40)

2. a) Show that $\arg\left(\dfrac{z-1}{z+1}\right)=\dfrac{\pi}{4}$ represents a circle.

   b) Prove that the necessary condition for a function f(z) = u(xy) + iv(xy) to be analytic is $u_x=v_y$ and $u_y=-v_x$.

OR

P.T.O.

3. a) Evaluate $\lim_{z\to 1+i}\left[\dfrac{z^2-z+1-i}{z^2-2z+2}\right]$.

b) Show that $f(z) = ze^z$ is analytic.

4. a) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

b) Find the orthogonal trajectories of the family of curves $2e^{-x}\sin y + x^2 - y^2 = c$.

OR

5. a) If $f(z) = u + iv$ is analytic and ϕ is any differentiable function of x and y, show that $\left(\dfrac{\partial\phi}{\partial x}\right)^2+\left(\dfrac{\partial\phi}{\partial y}\right)^2=\left[\left(\dfrac{\partial\phi}{\partial u}\right)^2+\left(\dfrac{\partial\phi}{\partial v}\right)^2\right]|f'(z)|^2$.

b) Show that $u = x^3 - 3xy^2$ is harmonic and find its harmonic conjugate.

6. a) Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)\,dx+(2y-x)dy$ along

i) The curve $y = x^2 + 1$.

ii) The line joining (0, 1) and (2, 5).

b) State and prove fundamental theorem on algebra.

OR

7. a) Evaluate $\int_C \dfrac{\sin(\pi z^2)+\cos(\pi z^2)}{(z-1)\,(z-2)}dz$ where C is a circle $|z| = 3$.

b) State and prove Cauchy's integral theorem.

8. a) Prove that the Bilinear transformation preserves the cross ratio.

b) Discuss the transformation $w = z^2$.

OR

9. a) Find the bilinear transformation which maps $z = 0, -i, -1$ on to $w = i, 1, 0$ respectively.

b) Show that the transformation $w=\dfrac{i-z}{i+z}$ makes the x-axis of the z-plane on to a circle $|w| = 1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.

PART – C

Answer **two full** questions. **(2×10=20)**

10. a) Find the root of the equation $x^3 - 4x + 1 = 0$ over (0, 1) by Regula-Falsi method.

    b) Find the cube root of 24, correct to three decimal places by Newton-Raphson method.

OR

11. a) Solve the equation

    $x + y + 54z = 110$

    $27x + 6y - z = 85$

    $6x + 15y + 2z = 72$ by Gauss-Seidel method.

    b) Find the largest eigen value of the matrix and its corresponding eigen vector $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ by power method.

12. a) Find the solution of $\frac{dy}{dx} = xy$ with $y(2) = 2$ at $x = 2.1$ correct to four decimal places, using Taylor series.

    b) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0) = 1$ for $x = 0.1$ by Euler's method.

OR

13. a) Solve $\frac{dy}{dx} = x + y$ with $y(0) = 1$ for $x = 0.1$ using Euler's modified method.

    b) Solve $\frac{dy}{dx} = xy$ given $y(1) = 2$ at $x = 1.2$ by Runge-Kutta method.

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